A Suggested Method for Computing Colorimetric Densities

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A paper presented at the 1991 Annual Technical Conference of the Technical Association of the Graphic Arts, Henrietta, NY, 5 - 8 May, 1991

Cite This Paper: J A Stephen Viggiano and C Jeffrey Wang, A Suggested Method for Computing Colorimetric Densities, *1991 TAGA Proceedings,* p 196 - 215.

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ABSTRACT

The colorimeter holds tremendous promise for color measurement in the Printing Industry. Yet, the measurements provided by the colorimeter are different from those provided by the more traditional instrument, the densitometer, making interpretation of colorimetric measurements (in the context of the industry) difficult. This is a significant obstacle to the acceptance of the colorimeter as a pressroom tool.

Simple conversion of the X, Y, Z tristimulus values into densities is not satisfactory for most purposes. The Preucil coordinates of Hue and Grayness, which are used in color correction, are distorted when using colorimetric densities computed directly from X, Y, and Z.

This paper discusses the transformation of the XYZ tristimulus values to an RGB-space. The transformation was selected to provide close agreement with Status T measurements, while enclosing most of the colors apt to be encountered in reproduction work. The new coordinates may be used directly in the computation of such parameters as Preucil Hue and Grayness. Transformation of these coordinates into a uniform color space, such as CIELAB or CIELUV, is both simple and exact.

INTRODUCTION

The densitometer is the most widely used instrument for performing color measurement-type tasks in our industry. Typically, measurements are made through Red, Green, and Blue filters, which are used to evaluate inks for the following attributes:

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- Ink Strength (Solid Ink Density)
- Ink Hue / Hue Error
- Ink Grayness
- Trapping
- Effective Relative Absorptance (Murray-Davies)
- Halftone Dot Area (Yule-Nielsen)

The first three attributes relate directly to color correction and masking. Together with three graphical tools (which will be discussed later) they form the Preucil system of ink evaluation. [1] [2] This system has proven extremely useful in producing high-quality process color reproduction. The general task of ink evaluation may be used, in other words, to get a reproduction to look as much like the original as is possible (or desired), subject to the constraints imposed by the differences in the original and reproduction gamuts. For such purposes, it is highly desirable to use a system which is related to human color perception.

Unfortunately, the specie of reflection densitometry most often used for these tasks, ISO Status T, is not visually referenced. Status T is intended, "[t]o evaluate the modulation produced by an image from which a colour separation process is to be performed in preparation for a three-colour process." [3] The Status T densitometer is intended to be related to the spectral response of a color separation system, rather than that of the human eye.

Nevertheless, Status T and its historical antecedents have served the industry nobly for decades. It could be argued, then, that it would be desirable for a set of colorimetric densities to correspond closely with Status T measurements.

ANALYSIS OF AN INK SET USING THE PREUCIL SYSTEM

In order to illustrate an important application of Densitometry and the Preucil system, we shall evaluate a set of process inks. These calculations could also be used to compute color correction requirements, but we shall not go into these details here.

In Table 1 are a set of measurements of a set of inks for sheetfed offset press proofing. Status T densities are reported relative to the paper, so the densities recorded for the paper are all zero. Also reported are the values for Preucil Hue Error, Grayness, and Strength. (No values for the Preucil parameters are reported for the paper.)

	Dr	Dg	Db	Hue Error	Gray-ness	Strength
Paper	0.00	0.00	0.00	—	—	—
Cyan	1.05	0.27	0.09	0.19	0.09	1.05
Magenta	0.16	1.05	0.53	0.42	0.15	1.05
Yellow	0.01	0.06	0.75	0.07	0.01	0.75

Table 1.
Analysis of an Ink Set using Status T Densitometry and the Preucil System.

It is customary, as we have done in this example, to use a subscript in conjunction with densities in order to indicate the band to which they correspond. For example, the symbol D_r denotes a density which applies to a Red band.

We would like to draw attention to an additional aspect of the data which appear in Table 1. Note that the densities of the Yellow ink are ordered (smallest to largest) Dr, Dg, and Db. This is natural, as the Yellow ink primarily absorbs Blue light, and a small amount of Green light (in a narrow spectral transition region). Thus, one would conclude that the Yellow ink deviates in hue (if only slightly) away from Green, and towards Red.

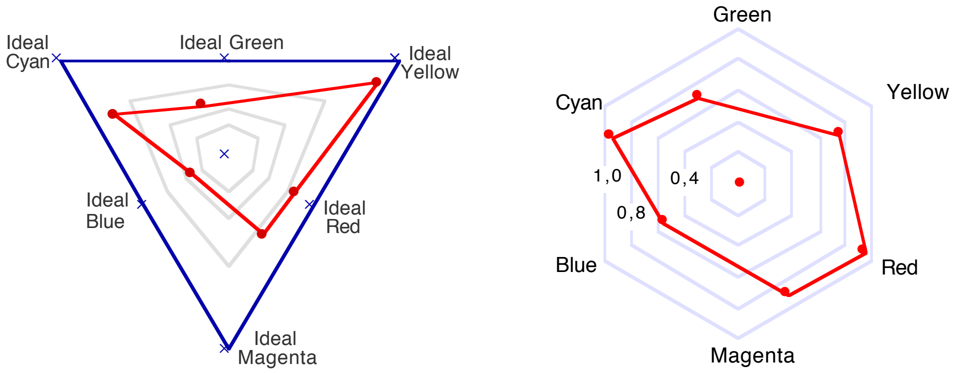

Figure 1.
Preucil Triangle and Hexagon for the Sample Inks measured with Status T Densitometry. The light gray lines in the interior of the Triangle indicate (outer to inner) Grayness Levels of 20, 40, and 60 percent.

Figure 1 depicts the Preucil Triangle and Hexagon for these inks. (In plotting these diagrams, we have also included the overprints, although their densities do not appear in Table 1.) The corners of the Triangle correspond to Cyan, Magenta, and Yellow inks that have neither Hue Error nor Grayness. Such inks are referred to as "Ideal" inks in the Preucil system. (Such inks are also impossible to achieve in practice, at least for Cyan and Magenta.) The light gray lines in the interior of the triangle correspond to grayness levels of 20, 40, and 60 percent, from the outermost to the innermost.

Ideal Hues for the inks and their overprints appear as the vertices of the gray hexagons on the Hexagon diagram. Each gray hexagon corresponds to a level of strength. The inner hexagon indicates a strength of 0.20 density units, while the outer hexagon indicates a strength of 1.00 density units. (Strength values can and do exceed 1.00 density units; additional hexagons are added in such situations.)

Under the Preucil system, the Gamut, or range of colors which these inks can produce, is indicated by the lines connecting the three inks and their overprints. The Triangle indicates the gamut in terms of Hue and Grayness, while the Hexagon indicates the gamut in terms of Hue and Strength.

DIFFERENCES BETWEEN DENSITOMETRY AND COLORIMETRY

The fundamental difference between Densitometry and Colorimetry is that the Tristimulus Values (the basic units of Colorimetry) are directly proportional to the amount of light reflected from, transmitted by, or emitted from an object, while Density units (the basic units of Densitometry) are logarithmically related. Essentially, this is a very small difference; both linear and logarithmic units could be reported by a modern, computerized instrument.

John Yule often argued the near triviality of this difference. In a 1951 paper entitled, "Colorimetric Investigations in Multicolor Printing," he and R. Colt write: [4]

> It may be noticed that the title of this paper refers to colorimetric investigations, although tristimulus values and trichromatic coefficients have not even been mentioned. Actually, these investigations apply equally well to tristimulus values. In fact, the Red, Green, and Blue densities in the tables would have been tristimulus densities if the densitometer happened to have spectral responses corresponding to the I.C.I. *[now CIE]* standard observer.

Together with Milt Pearson, he wrote, "In distinguishing colorimeters from densitometers, we are not concerned with the fact that densitometers read in density units since it is easy to convert from one type of unit to another." [5] The actual equations for converting tristimulus values to densities appear in later sections of this paper.

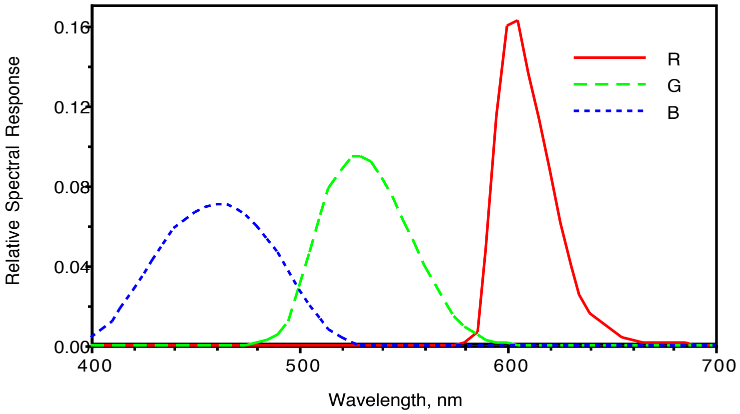

Figure 2.
The Spectral Products (relative spectral sensitivity) curves for Status T densitometry.

This mathematical detail is the only *fundamental* difference between densitometry and colorimetry. There is, however, a very important *practical* distinction between the two. The spectral sensitivity of a given densitometer channel is referred to as its *spectral product.* These spectral products determine, to a very large degree, the response of each channel of the instrument to colored objects. The spectral products for Status T densitometry appear in Figure 2.

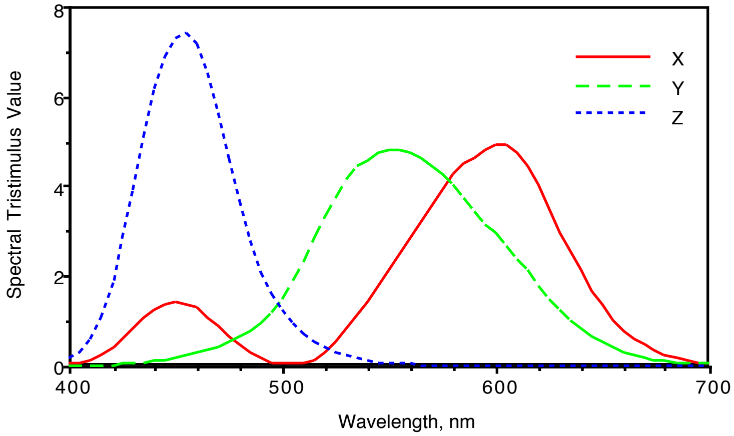

Figure 3.
The Spectral Tristimulus Values of Illuminant D50. These curves are the "spectral products" of the colorimeter. Note that the X-curve has a small second hump in the 400 – 500 nanometer region.

The spectral products of colorimeters differ from those of Status T densitometers in that they are related to those of the eye. Figure 3 illustrates the spectral products (or, as they are called in colorimetry, the spectral tristimulus values) of a colorimeter using both the CIE 1931 Standard Observer and Illuminant D50. These are the conditions which are best suited for colorimetry of domestic process color reproductions.

Both diagrams have three curves. In both Figures 2 and 3, there are curves which weight each of the three major spectral regions (Red, Green, and Blue). One of the salient differences between the two sets of sensitivity curves is their width and shape. All three of the curves for Status T densitometry have a single maximum; while the X-curve for colorimetry has a large maximum in the Red region, and a small local maximum in the Blue region. Further, the Status T Red and Green curves are narrower than the CIE X and Y curves. In other words, the Status T curves are more narrow band (in the Red and Green regions) than the CIE curves.

DIRECT TRANSFORMATION OF XYZ TRISTIMULUS VALUES INTO DENSITIES

We would like to make a small disclaimer here. Some of the ensuing discussion might seem to be an indictment of the CIE XYZ system of colorimetry. *This is not the case.* The authors use the XYZ system extensively, and feel that it is an indispensible tool. We shall, however, illustrate some problems which arise when this system is applied in an inappropriate manner. That the illustrations of these problems is not an attack on the XYZ system of colorimetry is further underscored by our use of it in our solution.

The usual output of a colorimeter is a set of XYZ tristimulus values. Why not simply convert these tristimulus values directly into densities, using the simple conversion to which Pearson and Yule alluded? This certainly is the most direct means of obtaining colorimetric densities. The equations to perform this conversion are as follows:

$$\begin{aligned} D_x &= \mathrm{Log}(X_n) - \mathrm{Log}(X) \\ D_y &= \mathrm{Log}(Y_n) - \mathrm{Log}(Y) \\ D_z &= \mathrm{Log}(Z_n) - \mathrm{Log}(Z) \end{aligned} \tag{1}$$

where X, Y, and Z are the tristimulus values of the object whose densities are to be computed;

Xn, Yn, and Zn are the tristimulus values of the nominally white object; and

Dx, Dy, and Dz are the resulting colorimetric densities.

Often, the paper is considered the nominally white object. Using the tristimulus values of the paper as Xn, Yn, and Zn will produce relative or "zeroed on paper" densities, which are often used with Status T measurements. If, however, absolute densities are desired, the tristimulus values of the perfect reflecting diffuser should be used instead.

COMPARISON OF STATUS T AND XYZ DENSITIES

The X tristimulus value has most of its sensitivity in the Red region of the spectrum, so its associated density, Dx, would be expected to compare most closely with the Red density, Dr. Similarly, the bulk of the sensitivity of the Y tristimulus value is in the Green region of the spectrum, so Dy would be expected to correspond most closely to the Green density, Dg. Finally, because nearly all of the sensitivity of the Z tristimulus value is in the Blue region of the spectrum, one would expect Dz to compare favorably with the Blue density Db.

	X	Y	Z	Dx	Dy	Dz
Paper	70.58	73.32	56.35	0.00	0.00	0.00
Cyan	16.94	25.66	45.51	0.62	0.46	0.09
Magenta	31.86	17.52	17.62	0.35	0.62	0.50
Yellow	60.06	64.64	9.11	0.07	0.05	0.79

Table 2.
Colorimetric Measurements (Tristimulus Values) and computed Tristimulus Densities Dx, Dy, and Dz of the same inks analyzed in Table 1. Note that the densities reported for the paper are all zero.

In Table 2 are colorimetric measurements of the same inks evaluated densitometrically in Table 1. In addition to the XYZ tristimulus values we also present their corresponding densities. Note that the densities for the paper are all zero. This is because we have used the paper as the nominally white stimulus. The tristimulus values of the paper are thus used as Xn, Yn, and Zn in the calculation of the densities Dx, Dy, and Dz in Equation (1).

Recall our earlier discussion on the ordering of the densities for the Yellow ink. While with the Status T densities the order (smallest to largest) was Dr, Dg, Db, with the colorimetric densities the order of the "Red" and "green" densities is reversed. Thus, under this system, it might appear that the hue of the Yellow ink favors the Green, rather than the Red, as was the case with the Status T measurements.

This is not the case, because the primaries of the XYZ system are not Red, Green, and Blue. The X primary is actually reddish-purple in hue. (The Y primary is a slightly yellowish Green, and the Z primary is, in fact, Blue in hue.) In addition, these XYZ primaries are said to be imaginary; they are much more saturated than colors which can be produced in real life. Thus, when using the colorimetric densities Dx, Dy, and Dz, the "ideal" colors on a Preucil diagram will not only be slightly off in hue, but will also be much more saturated than any real color ever could be. The net effect is that when inks are evaluated using this system, the hues will be distorted, the grayness values will tend to be too large, and the strength values will tend to be too low.

Table 3 contains a comparison of the two sets of densities measured / computed from the same set of three inks, for comparison purposes. There seems to be fairly good agreement between Dz and Db. These two density components compare well for all three inks. The largest difference between the two is only 0.04 density units.

	Dr	Dx	Dg	Dy	Db	Dz
Cyan	1.05	0.62	0.27	0.46	0.09	0.09
Magenta	0.16	0.35	1.05	0.62	0.53	0.50
Yellow	0.01	0.07	0.06	0.05	0.75	0.79

Table 3.
The Status T and Colorimetric Densities Dx, Dy, and Dz are compared. There is fairly close agreement for the Blue – Z band, but in the Red – X and Green – Y bands the two sets of densities differ greatly.

Unfortunately, the comparison between Dg and Dy is not nearly as good. While the smallest difference is only 0.01 density units, the largest such difference is 0.43, a very significant difference. The same large maximum difference applies to the Dr - Dx comparison; there the minimum difference is 0.06 units. Clearly, this method of computing densities produces results which differ greatly from those of Status T.

The disparity between these two sets of densities is further elucidated by the comparison of the Preucil parameters. Table 4 contains two sets of Preucil parameters — one set computed from the Status T densities, the other from the colorimetric densities Dx, Dy, and Dz.

	Hue Error		Grayness		Strength	
	ST	XYZ	ST	XYZ	ST	XYZ
Cyan	0.19	0.69	0.09	0.15	1.05	0.62
Magenta	0.42	0.58	0.15	0.56	1.05	0.62
Yellow	0.07	0.03	0.01	0.07	0.75	0.79

Table 4.
The Preucil Parameters of Hue Error, Grayness, and Strength are used to illustrate the disparity between the two sets of measurements. In the legend, "ST" refers to the Status T densities, and "XYZ" refers to the tristimulus densities Dx, Dy, and Dz.

The Yellow ink seems to have fared the best. The strength values for Yellow match, and the other values seem to agree fairly well. However, the difference in hue is larger than indicated in the table: the two hue errors are actually in opposite directions; the Status T value is towards Red, while the value according to the XYZ system is towards the Green.

The comparison for the other inks is less favorable. Both Cyan and Magenta have much lower strengths under the XYZ system of densitometry. The Hue Error values are very different for Cyan, while the Grayness values differ greatly for the Magenta ink. Clearly, this is not a desirable situation.

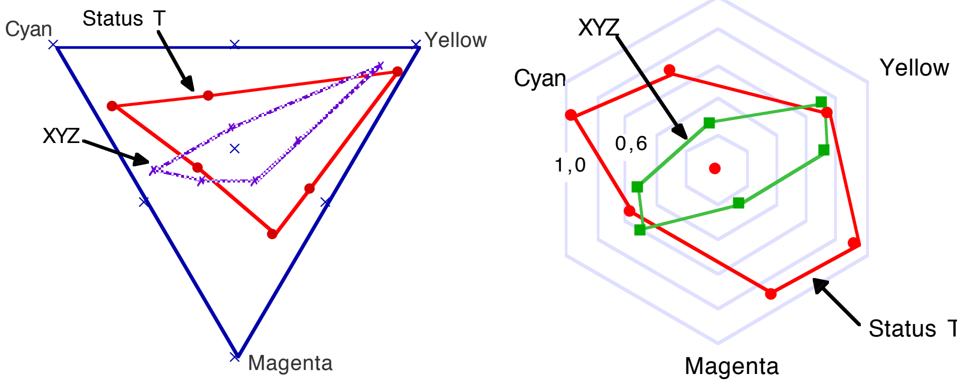

Figure 4.
This figure is a graphical comparison of the two "families" of densities — Status T and XYZ — using both the Preucil Triangle and Hexagon. The gamut of this ink set is attenuated when the XYZ system is used.

TRANSFORMATION OF TRISTIMULUS VALUES

One way in which the problems with the densities Dx, Dy, and Dz may be mitigated is to first transform X, Y, and Z into different tristimulus values, which we shall call R, G, and B. (Under a principle known as the Ives–Luther condition, [6] [7] any linear combination of X, Y, and Z will also be a tristimulus value.) These new tristimulus values can then be converted into densities, as was done for the original tristimulus values using Equation (1).

Because these new tristimulus values are computed from the original XYZ tristimulus values, the new system will still be colorimetric in nature. Transformation of the new values into uniform color spaces, such as CIELUV and CIELAB, will be simple and exact.

One such transformation was suggested by Warren Rhodes during a discussion at the 1987 TAGA Conference in San Diego, after a presentation by one of the authors on computing colorimetric densities. [8] The transformation suggested by Rhodes was MacAdam's Narrowest Color Mixture Curves without negative portions. [9] This transformation of XYZ was unsatisfactory; the Grayness values were still too great, although the values of Hue Error were improved slightly. If plotted on one of the Preucil diagrams, the results would appear much more like Figure 1 than Figure 4.

OUR EARLIER SUGGESTION

The method suggested in the 1987 paper also proved unsatisfactory, but for an entirely different reason. There was reasonably good agreement between the colorimetric densities and Status T densities for the small sampling of materials evaluated. Unfortunately, it failed when presented with CRT data in our RGB to CMYK transformation work. Some of the transformed tristimulus values were negative, rendering them unsatisfactory for conversion into densities. In addition, computed hues were still disturbingly different from those computed from Status T measurements.

The reason for this failure is because some of the objects we tried to evaluate fall outside, or close to, the boundaries of the RGB-primary gamut. In order to avoid negative and complex densities, the RGB primaries must enclose all the colors, on a chromaticity diagram, which you will encounter. The primaries suggested in 1987 do not.

This requirement of enclosing as many colorants as possible is in direct competition with our desire that the colorimetric measurements should closely match Status T. As was pointed out in the 1987 paper, the projection of the Status T spectral products onto color matching vector space produced primaries that were too narrow. This is primarily because the Status T Red spectral product is fairly narrow in bandwidth, as can be seen in Figure 1.

OUR NEW APPROACH

The first step in our new approach was to compile a database of colored objects which are routinely used in graphic reproduction. While these are mostly inks, it is important to realize that the densities of original objects, such as photographic transparencies and color CRT monitors, must be considered, as well. The colored materials we considered in this study were:

- Process Color Inks
- Spot Color Inks (PMS® colors)
- Color Proofing Materials
- Color Transparency Materials
- CRT Phosphors

The database included the u', v' chromaticity coordinates of these objects. In addition, the Status T densities, relative to the paper, of the Process Color inks were included, for comparison purposes. The chromaticity coordinates would determine the gamut which needed to be enclosed; while the Status T measurements would allow us to evaluate our results.

Our approach was to select an RGB gamut as narrow as possible, while providing good agreement with the Status T measurements.

METHODOLOGY

We discuss here the colorants included in the database:

Process Color Inks: As was pointed out in the 1987 paper, it is sufficient to consider, for any given set of inks, the three process inks and their two-color overprints. It is not necessary, for this task, to consider the three-fold infinity of colors which can be produced by the three inks in combination. This is because only the chromaticity coordinates need to be considered. Under the simplifying assumption of the Neugebauer model, the chromaticities which can be produced by the three inks will occupy an irregular hexagon whose vertices are the chromaticity coordinates of the three inks and their two-color overprints.

Four complete sets of process color inks were included in our database. These are:

- A representative set of SWOP proofing inks
- A set of Commercial Sheetfed inks
- A set of Commercial Heatset Web inks
- A set of Japanese Commercial inks

Also included in the database were the Cyan, Magenta, and Yellow inks of CEI 30–89, which are representative of an emerging new European standard. The data for these inks were kindly provided by Dr K. Schläpfer of EMPA / UGRA in

Switzerland. The Red, Green, and Blue overprints were not included in this set. The individual colors were not very different from those of other sets, however, so it was felt that the RGB overprints would be adequately represented by those of the other ink sets.

Spot Color Inks: Most spot colors appear in the *Pantone® Color Formula Guide,* which consists of mixtures of several basic colors. All nine of the "Basic Colors II" were included in the database, together with 12 fully saturated admixtures. None of the metallic or fluorescent inks in the *Pantone Color Formula Guide* were included.

Color Proofing Materials: Samples of the following prepress color proofing materials were included in the database:

- 3M Matchprint®
- DuPont Chromalin®
- Kodak Signature®

All ink sets (with the exception of the European ink set, which was provided in spectral form) and proofing materials were measured with a Gretag SPM-100 spectrophotometer. Absolute measurements were taken; these were converted to relative measurements off-line using the spreadsheet program that also served as the database manager. Both densities and tristimulus values were measured for the process ink sets and proofing materials; tristimulus values only were recorded for the spot color inks.

Color Transparency Materials: Originals are often supplied in this form; it is very important to consider these media so that color correction may be performed. Individual spectral dye density curves were obtained for two popular films, Kodak Kodachrome® and Ektachrome®.

We felt that a limiting gamut for these materials could be obtained by considering these dyes, both individually and in combination, to Equivalent Neutral Densities of up to 2.50. While it is common to obtain densities of 3.00 from modern transparency material, exposure punch-through and inter-image effects both will reduce significantly the saturation obtainable at large ENDs.

It is tempting to point out that other photographic materials, including those from other manufacturers, incorporate a yellow dye which is very different from the yellow dyes used in the two materials mentioned above. While this is true, it does not invalidate our results because our RGB primaries include all possible yellows within their gamut.

CRT Phosphors: We included the chromaticity coordinates of these sets of RGB phosphors in our database:

- NTSC (US) Standard
- PAL (European System I) Standard
- EIA – P22 (Six Variations)
- SMPTE – C
- Graphic Arts Long Persistence

Because the CRT operates on an additive principle, any set of primaries which enclose the RGB phosphor chromaticities will enclose any chromaticity they can produce in combination.

The P-22 phosphor data were obtained from an EIA publication. [10] The Graphic Arts Long Persistence phosphors were measured directly in our laboratory. Other phosphor data were obtained from a standard source on the color science of television systems. [11]

RESULTS

The chromaticity coordinates of all colorants in the database are plotted in Figure 5. While the CRT phosphors provide some definite corners, it was also important to be mindful of the photographic dyes (particularly the Magenta – Yellow combination) and of the Spot colors.

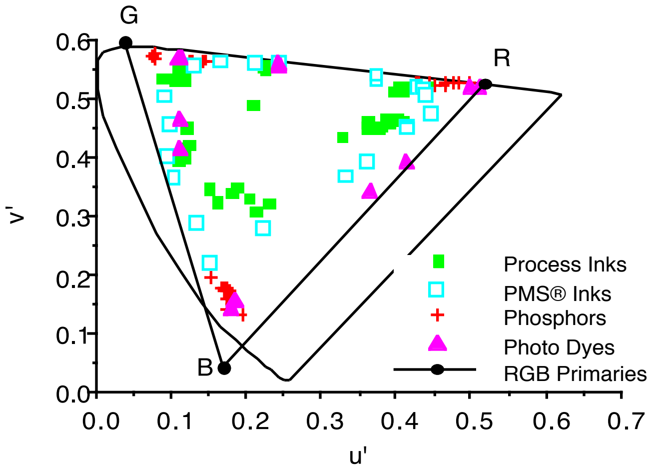

Figure 5.

The chromaticity coordinates of the colorants in the database are plotted on the 1976 u', v' diagram. The gamut of the system, for which no reflectances will be negative, is indicated by the triangle connecting the RGB primaries, which also appear on the diagram. It is important for the other points to be enclosed within this triangle.

Based on these measurements, the following chromaticities, which enclose all the colorants in the database, were selected:

	x	y	u'	v'
Red	0,6920	0,3087	0,5202	0,5222
Green	0,1328	0,8790	0,0400	0,5956
Blue	0,1236	0,0129	0,1700	0,0400

These primaries are also plotted on Figure 5, as are lines connecting them. In order to avoid negative reflectances, it is important that the primaries enclose all of the items in the database.

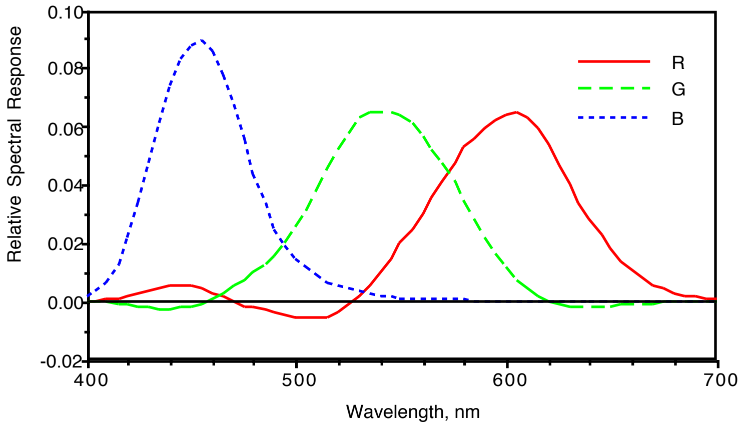

Figure 6.
Spectral Products for the New Method.

In addition to these chromaticity data, the mapping of the XYZ white point into the RGB-space is needed to calculate the transformation. Under standard graphic arts conditions (illuminant D50 and 1931 Standard Observer) the XYZ white point is X = 96.40, Y = 100.00, Z = 82.46. The logical white point for the RGB system is R = 1.00, G = 1.00, B = 1.00. Using the technique outlined in the Appendix, the following transformation equations were derived:

$$
(2) \qquad
\begin{aligned}
R &= 0.01 \cdot (\;1.4391\,X - 0.2202\,Y - 0.2027\,Z) \\
G &= 0.01 \cdot (-0.7310\,X + 1.6386\,Y + 0.0801\,Z) \\
B &= 0.01 \cdot (-0.0064\,X + 0.0171\,Y + 1.1995\,Z)
\end{aligned}
$$

where R, G, and B are the Red, Green, and Blue tristimulus values.

The transformation defined in Equation set (2) has spectral products with small negative lobes. (See Figure 6.) These small negative lobes preclude the production of an instrument with these spectral products. The RGB values may be computed by tristimulus integration of spectral data, or may be computed from XYZ measurements using Equation set (2).

These RGB tristimulus values may be transformed into densities in precisely the same manner as the XYZ tristimulus values were in Equation set (1):

$$
\begin{aligned}
& D_r = \mathrm{Log}(R_n) - \mathrm{Log}(R) \\
(3) \qquad & D_g = \mathrm{Log}(G_n) - \mathrm{Log}(G) \\
& D_b = \mathrm{Log}(B_n) - \mathrm{Log}(B)
\end{aligned}
$$

where Rn, Gn, and Bn are the RGB tristimulus values of the nominally white object.

	R	G	B	Dr	Dg	Db
Paper	0.740	0.731	0.684	0.00	0.00	0.00
Cyan	0.095	0.333	0.549	0.89	0.34	0.10
Magenta	0.384	0.068	0.212	0.28	1.03	0.51
Yellow	0.704	0.627	0.116	0.02	0.07	0.77

Table 5.
RGB measurements transformed from the XYZ Tristimulus values presented in Table 2. The Paper is used as the nominally white object.

Table 5 contains the R, G, B tristimulus values of the example inks, transformed from the XYZ tristimulus values in Table 2 using Equation 2. Also included are the corresponding densities; these were calculated from the RGB tristimulus data using Equation (3).

These new densities do compare more favorably with the Status T measurements. Table 6 contains a side-by-side comparison.

	Red Densities		Green Densities		Blue Densities	
	ST	New	ST	New	ST	New
Cyan	1.05	0.89	0.27	0.34	0.09	0.10
Magenta	0.16	0.28	1.05	1.03	0.53	0.51
Yellow	0.01	0.02	0.06	0.07	0.75	0.77

Table 6.
The densities as measured by Status T and computed using the new method are presented side-by-side. "New" denotes those densities computed using the new technique.

DISCUSSION

Of the three density components, the Red did the worst. There is a difference of 0.16 density units for the Cyan ink in this band. The problem encountered was the tradeoff mentioned earlier between our desire for narrow sensitivities and a wide RGB gamut. While the primaries could have been adjusted to provide better agreement in the Red channel, some of the important objects (particularly the CRT phosphors and color transparency materials) would have been forced out of the gamut. We felt that this is a reasonable compromise.

The Green densities are in much closer agreement. A maximum deviation of only 0.07 units exists between the two sets of Green densities. In the Blue band, the maximum difference is only 0.02 units.

Figure 7 shows the suitability of the new transformation. Both the Preucil Triangle and Hexagon show dramatic improvement over the results shown in Figure 4, where the XYZ densities are compared to Status T.

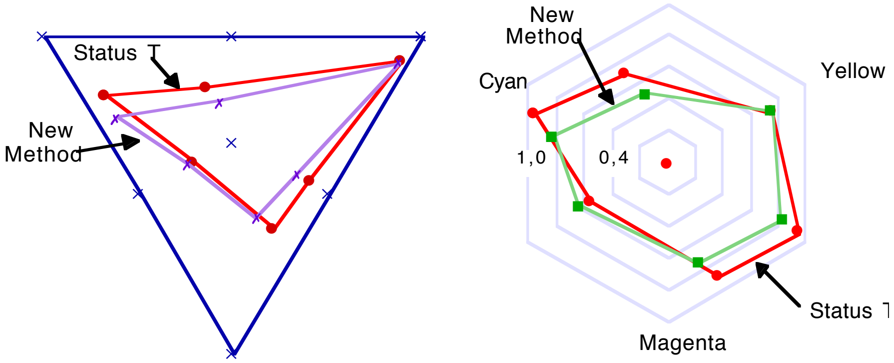

Figure 7.
The new method of computing densities is compared graphically to Status T, using both the Preucil Triangle and Hexagon diagrams.

CONVERSION OF DENSITIES TO UNIFORM COLOR COORDINATES

We had mentioned earlier that the densities computed by this method could be converted into Uniform Color Space (*eg.,* CIELUV, CIELAB) coordinates. This is accomplished by converting the densities back into RGB tristimulus values:

$$
(4) \qquad
\begin{aligned}
R &= R_n \bullet 10^{-D_r} \\
G &= G_n \bullet 10^{-D_g} \\
B &= B_n \bullet 10^{-D_b}
\end{aligned}
$$

and then converting the RGB tristimulus values into XYZ tristimulus values, using Equation set (5):

$$
(5) \qquad
\begin{aligned}
X &= 74{,}57\,R + 9{,}90\,G + 11{,}94\,B \\
Y &= 33{,}27\,R + 65{,}48\,G + 1{,}25\,B \\
Z &= -0{,}07\,R - 0{,}88\,G + 83{,}41\,B
\end{aligned}
$$

These XYZ tristimulus values are converted into CIELUV, CIELAB, etc., coordinates in the usual way.

We may use this result to compute the CIELUV coordinates of an ideal ink. For example, a Yellow ink with a strength of 1.30 will have a Blue density of 1.30, while the Red and Green densities will both be zero. By application of Equation Set (4), we obtain RGB tristimulus values of 1.0000, 1.0000, and 0.0501, respectively. Via Equation Set (5), we compute the XYZ tristimulus values, which are 85.07, 98.81, and 3,23, respectively. These may be converted into CIELUV coordinates of $L^* = 99.5$, $u^* = 8.6$, and $v^* = 98.1$.

The same XYZ tristimulus values may be converted into CIELAB coordinates; the results are $L^* = 99.5$ (as before), $a^* = -18.4$, and $b^* = 131.3$. We would like to caution the reader that such an ink is physically impossible to produce.

ACKNOWLEDGEMENTS

The authors would like to express their sincere appreciation to the Imaging Division of the RIT Research Corporation for the opportunity to prepare this paper. We also owe thanks to Yan Liu for his assistance with the many measurements.

We dedicate this paper to Mr Frank Preucil, whose work on color correction theory has meant so much to the Printing Industry.

APPENDIX

CALCULATION OF TRANSFORMATION EQUATIONS

The transformation from XYZ to RGB tristimulus values may be specified by a set of chromaticity coordinates for the RGB primaries, and the tristimulus values (in both systems) of a white point. (Throughout this process, it is important to distinguish between tristimulus values, which are represented with capital letters, from chromaticities, which are represented with lower-case letters.)

The first step is to gather the xyz chromaticity coordinates of the RGB primaries into a matrix. The x-chromaticities go into the first row, the y-chromaticities into the second, and the z-chromaticities into the third:

$$(A1) \qquad C = \begin{bmatrix} x_R & x_G & x_B \\ y_R & y_G & y_B \\ z_R & z_G & z_B \end{bmatrix}$$

(The z-chromaticity is simply 1 – x – y.) Next, the tristimulus values of the white must be specified in both systems. In the XYZ system, using Illuminant D50 and the 2-degree Standard Observer, $X_W = 96.40$, $Y_W = 100.00$, and $Z_W = 82.46$. In the RGB system, we use $R_W = G_W = B_W = 1.00$.

The tristimulus sums S_R, S_G, and S_B are computed next:

$$(A2) \qquad \begin{bmatrix} S_R \\ S_G \\ S_B \end{bmatrix} = \begin{bmatrix} 1/R_W & 0 & 0 \\ 0 & 1/G_W & 0 \\ 0 & 0 & 1/Z_W \end{bmatrix} \cdot C^{-1} \cdot \begin{bmatrix} X_W \\ Y_W \\ Z_W \end{bmatrix}$$

The matrix of the XYZ tristimulus values of the RGB primaries is then:

$$(A3) \qquad T = C \cdot \begin{bmatrix} S_R & 0 & 0 \\ 0 & S_G & 0 \\ 0 & 0 & S_B \end{bmatrix}$$

The relationship between XYZ and RGB is defined by this matrix equation:

$$(A4) \qquad \begin{bmatrix} R \\ G \\ B \end{bmatrix} = T^{-1} \bullet \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

For an alternate discussion of this technique, see, for example, Hunt's Third Edition. [12]

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