The Comparison of Radiance Ratio Spectra:
Assessing a Model's "Goodness of Fit"

R · I · T Research Corporation

A presentation made at SPSE's 43rd Annual Conference
20 – 25 May 1990 • Rochester, New York

J A Stephen Viggiano • Imaging Division

Cite this Paper: Viggiano, J A Stephen, The Comparison of Radiance Ratio Spectra: Assessing a Model's "Goodness of Fit". *Advanced Printing of Conference Summaries: SPSE's 43rd Annual Conference,* 1990. Springfield, VA: SPSE – The Society for Imaging Science and Technology, p 222 – 225.

THE COMPARISON OF RADIANCE RATIO SPECTRA:
Assessing a Model's "Goodness of Fit"

J A STEPHEN VIGGIANO
RIT RESEARCH CORPORATION
ROCHESTER, NY 14623

INTRODUCTION

In the development of models which predict the color performance of some process, it is necessary to select some criterion which may be optimized by the model (and its condition-specific calibration). Some metric, or index, which indicates how well the model fits observed data, is also needed. For subtractive processes, the difference spectrum between the observed and predicted curves will in general contain a component which is illuminant metameric to black. This component will be ignored by a comparison of tristimulus values or CIELAB coordinates, although it bears on the degree of adequacy of the model. A more desirable method of comparison would not ignore this component.

This abstract discusses a spectral comparison index, which has proven useful as both an optimization criterion and evaluation metric.

Desirable Characteristics of a Comparison Index:

The desirable characteristics which a comparison method must possess are:

1. The index should be relatively simple to compute and interpret.

2. The index should produce numbers which relate well to subjective visual judgments of the adequacy of the match.

3. The index should be sensitive to components in the difference spectrum which are metameric to black.

Tacitly included in the first characteristic is the desire for a univariate (*i.e.*, single-dimensional) index. The second characteristic tacitly includes the condition that the index must equal zero when the difference spectrum is zero throughout the spectrum.

Comparison Indices In the Prior Art:

As mentioned above, the Total Color Difference (ΔE^*) between the spectrum as measured and as predicted by the model can be used as a comparison index, using a uniform color space, such as CIELAB or CIELUV. However, this approach does not satisfy Desirable Characteristic (2), above. One strategy for avoiding this disadvantage is to compute a suite of ΔE^* values, each computed under one of a variety of illuminants. [1] This, in turn, violates our desire for a univariate index, as implied in Desirable Characteristic (1).

Another possibility is to compute some norm of the difference spectrum. Judd and Wyszecki, in considering what is essentially (to within a multiplicative constant) the Root Mean Square difference, point out that ". . . all wavelengths of the visible spectrum (λ = 380 to λ = 780 nm) are given equal weights. This may overweight particularly the ends of the spectrum, which, as we know, are less important in color evaluations." [2] Unweighted norms, while fulfilling Desirable Characteristics (1) and (3) above, do not produce results which relate well to subjective evaluations of the adequacy of the fit.

We may obtain the advantages of both these schemes by combining them. Consider the following situation: The predicted curve matches the observed at every wavelength but one. In such a case, ΔE* meets all the Desirable Characteristics above. While this admittedly is a naive situation, it can be made more robust so that actual situations may be addressed.

An Index Based on a Spectrum of ΔE* Values:
Indicate the contingency of the ΔE* value computed above on the particular wavelength under consideration by denoting it as ΔE* (λ). By performing this operation for every wavelength for which we have data, a spectrum of ΔE* values is obtained. In computing these ΔE* values, the radiance ratio for the predicted curve is assumed to equal that of the observed curve, *except at the wavelength currently under investigation.* An obstacle to this fulfilling all three of our desirable characteristics is that it is multivariate. This obstacle is removed by computing a norm of these values, rather than the differences in spectral radiance ratio.

The first- and second-order norms of this ΔE* spectrum may be denoted:

$$P_1 = \sum_{\lambda} \Delta E^*(\lambda)$$

(1) $$P_2 = \sqrt{\sum_{\lambda} \left[\Delta E^*(\lambda)\right]^2}$$

Our work indicates a slight preference for the first norm, because it seems to have a closer relation to what would be expected from subjective comparisons of pairs of radiance ratio that have no metameric component. On the other hand, the second-order norm could be adapted better to least-squares optimization problems.

An Index Based on a Weighted Norm of the Difference Spectrum:
However, it is desirable to further simplify these norms. It would be easier to compute them, as well as use them for optimization criteria, if they could be cast as weighted norms of the difference spectrum. To this end, consider the following identity in the CIELAB color space:

(2) $$\Delta E^*(\lambda) \equiv \sqrt{\Delta L^{*2}(\lambda) + \Delta a^{*2}(\lambda) + \Delta b^{*2}(\lambda)}$$

which may be approximated by:

(2a) $$\Delta E^*(\lambda) \approx \left|\Delta\beta(\lambda)\right| \cdot i(\lambda)$$

where i (λ) is a weighting function, and is given by:

(3) $$i(\lambda) = \sqrt{\left(\frac{d L^*(\lambda)}{d\beta(\lambda)}\right)^2 + \left(\frac{d a^*(\lambda)}{d\beta(\lambda)}\right)^2 + \left(\frac{d b^*(\lambda)}{d\beta(\lambda)}\right)^2}$$

The first of these derivatives in (3) may be obtained from the chain rule:

(4) $$\frac{d L^*}{d\beta(\lambda)} = 116 \cdot k \cdot s(\lambda) \cdot \bar{y}(\lambda) \cdot \frac{d}{dY} f\left(\frac{Y}{Y_n}\right)$$

where

$$\frac{d}{dY} f\left(\frac{Y}{Y_n}\right) = \frac{1}{3Y} f\left(\frac{Y}{Y_n}\right) \quad \text{if } \frac{Y}{Y_n} > 0.008856$$

$$\frac{d}{dY} f\left(\frac{Y}{Y_n}\right) = \frac{7.787}{Y_n} \quad \text{if } \frac{Y}{Y_n} \leq 0.008856 \qquad (5)$$

For additional computational efficiency, as well as consistency, tabulated values of weighting factors for tristimulus integration [3] may be substituted for the factor $k \cdot s(\lambda) \cdot \bar{y}(\lambda)$ in Equation (4). These weighting factors are available in ASTM Publication E 308 – 85 for the 9 ASTM recommended illuminants, for both the 1931 and 1964 standard observers, and for 10- and 20 nanometer sampling intervals.

The other two derivatives in Equation (3) are given by:

$$\frac{d\,a^*(\lambda)}{d\,\beta(\lambda)} = 500\,k \cdot s(\lambda) \cdot \left[\bar{x}(\lambda) \cdot \frac{d}{dX} f\left(\frac{X}{X_n}\right) - \bar{y}(\lambda) \cdot \frac{d}{dY} f\left(\frac{Y}{Y_n}\right)\right] \qquad (6)$$

and

$$\frac{d\,b^*(\lambda)}{d\,\beta(\lambda)} = 200\,k \cdot s(\lambda) \cdot \left[\bar{y}(\lambda) \cdot \frac{d}{dY} f\left(\frac{Y}{Y_n}\right) - \bar{z}(\lambda) \cdot \frac{d}{dZ} f\left(\frac{Z}{Z_n}\right)\right] \qquad (7)$$

where the derivatives of the Pauli extension [4] to the cube root, f(*), are determined in analogy to Equation (5).

DISCUSSION

From the above derivation, it can be seen that the weights depend upon the tristimulus values of the observed spectrum. Unless the tristimulus ratio is small, the derivative df / dT (where T represents one of the tristimulus values) has an inverse relationship with the corresponding tristimulus value. This is consistent with the observations of Nimeroff and Yurow in their development of a metameric index. [5]

We have observed a much larger contingency of the magnitude of the weights on lightness, rather than hue or chroma. This has proven useful in solving statistical questions, such as sample-size determination. In such a case, a “Rule of Thumb” figure can be cited, based solely on the object’s lightness.

Table 1, on the next page, contains a sample set of weights, calculated using Illuminant D_{50}, the 1931 Standard Observer, a wavelength range of 380 - 730 nm, a wavelength interval of 10 nm., for a neutral object with an L* of 50. The individual components are shown in addition to their combined effect.

Weights for A Neutral Object (L* = 50)
Illuminant D-50, 1931 Standard Observer
X = 17.754 Y = 18.419 Z = 15.190

	(— Components —)			(Total)		(— Components —)			(Total)
WL	**L***	**a***	**b***	**i(λ)**	**WL**	**L***	**a***	**b***	**i(λ)**
380	0.000	0.021	-0.047	0.052	560	11.314	-18.551	19.414	29.138
390	0.000	0.064	-0.142	0.156	570	10.633	-7.738	18.283	22.521
400	0.002	0.310	-0.708	0.773	580	9.689	3.850	16.668	19.660
410	0.007	1.219	-2.764	3.021	590	8.157	14.114	14.039	21.513
420	0.027	4.021	-9.244	10.081	600	6.973	22.474	12.005	26.417
430	0.079	8.260	-19.484	21.163	610	5.677	26.105	9.778	28.449
440	0.193	12.268	-30.384	32.767	620	4.268	24.382	7.353	25.822
450	0.374	13.221	-35.906	38.264	630	2.918	19.074	5.029	19.941
460	0.614	10.707	-34.740	36.358	640	1.946	13.891	3.355	14.423
470	0.953	5.062	-26.554	27.049	650	1.175	8.869	2.026	9.173
480	1.480	-1.780	-15.687	15.857	660	0.681	5.296	1.174	5.467
490	2.197	-7.956	-6.513	10.515	670	0.374	2.955	0.645	3.048
500	3.521	-14.964	-0.082	15.373	680	0.189	1.505	0.325	1.551
510	5.533	-23.378	5.928	24.744	690	0.084	0.676	0.144	0.696
520	7.868	-30.746	11.727	33.834	700	0.042	0.338	0.072	0.348
530	9.923	-34.281	16.108	39.155	710	0.021	0.174	0.037	0.180
540	10.986	-32.405	18.453	38.876	720	0.010	0.076	0.016	0.079
550	11.528	-27.217	19.663	35.501	730	0.008	0.081	0.014	0.083

Table 1.
Spectral Weights (and Components) For a Medium Gray.

REFERENCES

1. Burningham, Norman W, and Roy S Berns, Analysis of color in electrophotographic images. *Advanced Printing of Conference Summaries. SPSE's 40th Annual Conference & Symposium on Hybrid Imaging Systems,* p 91.

2. Judd, Deane B, and Gunter Wyszecki, *Color in Business, Science and Industry,* 3rd edition. New York: Wiley, 1975. p 177 - 178.

3. *Standard Method for Computing the Colors of Objects by Using the CIE System.* Philadelphia: American Society for Testing and Materials, publication E 308 – 85, 1985. p 12 - 47.

4. Pauli, H., Proposed extension of the CIE recommendation on "Uniform color spaces, color difference equations, and metric color terms." *Journal of the Optical Society of America,* 1976. 66 : 8 : 866 - 867.

5. Nimeroff, I, and J A Yurow, Degree of metamerism. *Journal of the Optical Society of America,* 1965. 55 : 2 : 185 - 190.